ZFCρ Thermodynamics Paper V: The Channel-Normalized Winding Ratio — Dynamical Prediction of the Tsallis q Parameter in State-Coupled Oscillators

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Abstract

Thermodynamics Paper IV (DOI: 10.5281/zenodo.19605664) established the exact interpolation family $q = 1 + 1/K$ and listed "independently extracting K from dynamics to predict the Tsallis q" as its hardest testable prediction. The present paper realizes this prediction for the first time, on the Brusselator chemical oscillator.

The central finding is that K in a continuous-time oscillator is not the one-lag effective depth $m_{eff}(\tau)$ — which diverges as $1/\tau$ and is not an intrinsic parameter — but rather the channel-normalized winding ratio:

$$K_{dyn} = T / (n_{ch} \cdot \tau_{dec})$$

where T is the oscillation period, τ_{dec} is the radial decay time (both extracted from the damped-oscillation fit of the f-channel autocorrelation function), and $n_{ch} = 2$ is the number of forward/reset channels. The Tsallis q is then predicted as

$$q = 1 + n_{ch} \cdot \tau_{dec} / T$$

This formula contains zero free parameters and does not depend on any sampling step τ. Across a seven-point scan of the Brusselator bifurcation parameter b = 2.2–5.0, the formula yields MAE = 0.022 and a maximum deviation $|\Delta q| = 0.068$, substantially outperforming the fixed-τ proxy $m_{eff}(\tau)$ which collapses to $|\Delta q| = 0.215$ at b = 5.0.

The factor $n_{ch} = 2$ is not a fitted parameter. It is consistent with, and naturally explained by, the channel-averaged shielding conjecture of Thermo IV ($q = \Omega_{eff} / n_{ch}$): q measures per-channel shielding depth, so the total winding ratio T/τ_{dec} must be divided by the channel count.

The paper also reports two negative results that sharpen the scope of Thermo IV's open problem 2: (1) $m_{eff}(\tau) \propto 1/\tau$ is a representation effect, not an intrinsic layer count, in continuous-time systems; (2) the inverse participation ratio $K_{part} = 1/\Sigma w_j^2$ constructed from macroscopic ACF mode weights fails at finite βE because ACF mode weights are not microscopic shielding-layer weights. Neither negative result invalidates Thermo IV's mathematical results; both precisely narrow the conditions under which the $K \leftrightarrow m_{eff}$ bridge can hold.

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§1 Problem: How to Extract Thermo IV's K from Dynamics

1.1 Known and unknown

Thermo IV established two exact results:

$$e_q(-x) = \left(1 + \frac{x}{K}\right)^{-K}, \quad q = 1 + \frac{1}{K}$$

and the channel-averaged shielding conjecture:

$$q = \frac{\Omega_{\text{eff}}}{n_{\text{ch}}}$$

The interpolation family is an algebraic identity. The physical interpretation of K as an effective feedback order is a conjecture. Thermo IV listed "independently extracting K from the autocorrelation function to predict q" as prediction P1 — the hardest testable prediction [1].

The question is: how should K be measured in a continuous-time system?

Thermo II–III [2,3] defined the effective depth m_eff via $\eta \approx 1 - C(1)^{m_{\text{eff}}}$. If $K = m_{\text{eff}}$, then the same parameter simultaneously determines the dissipation rate η and the distribution shape q. This is Thermo IV's open problem 2.

1.2 Contributions

This paper makes four contributions:

(1) Negative result: $m_{\text{eff}}(\tau)$ is not an intrinsic quantity in continuous-time oscillators (§2).

(2) Negative result: Thermo IV's $q_{\text{eff}}^{(2)} = 1 + \Sigma\lambda^2$ does not apply to macroscopic ACF mode weights at finite βE (§3).

(3) Positive result: the channel-normalized winding ratio $K_{\text{dyn}} = T/(n_{\text{ch}} \cdot \tau_{\text{dec}})$ is identified as the operationally accessible dynamical counterpart of K in continuous-time oscillators (§4).

(4) Parameter scan: $q_{\text{pred}} = 1 + n_{\text{ch}} \cdot \tau_{\text{dec}} / T$ gives $|\Delta q| < 0.07$ across seven parameter points with zero τ-dependence (§5).

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§2 Negative Result I: $m_{\text{eff}}(\tau)$ Is Not Intrinsic

2.1 The τ-divergence of m_eff

On the Brusselator ($a = 1, b = 3, \sigma = 0.3$), using Thermo III's Protocol A ($f = a + x^2y$ for synthesis, $r = (b+1)x$ for degradation [8]) to extract η and C(1), then computing $m_{\text{eff}} = \log(1-\eta)/\log(C(1))$:

τ_steps	τ (time units)	η	C(1)	m_eff
2	0.010	0.195	0.999	143
5	0.025	0.194	0.994	36
10	0.050	0.193	0.981	11.2
20	0.100	0.188	0.943	3.5
50	0.250	0.165	0.817	0.9

η exhibits a plateau (0.19–0.20 from τ = 0.01 to 0.05), but m_eff spans two orders of magnitude from 143 to 0.9.

2.2 Mathematical inevitability

For any smooth continuous-time process, the short-lag retention satisfies C(1;τ) ≈ e^{−aτ}, so log C(1) ≈ −aτ, giving m_eff = log(1−η)/(−aτ) ∝ 1/τ.

This is not a numerical artifact. It is a representation effect: slicing the same physical feedback process at different step sizes naturally yields different step counts.

2.3 The accidental precision of fixed τ

At fixed τ_steps = 10, m_eff(b = 3.0) = 11.2, predicting q = 1 + 1/11.2 = 1.089. The measured q_fit(r^2) = 1.091. Deviation: +0.001.

However, this precision holds only near b ≈ 3.0. At b = 5.0, the deviation grows to 0.215, because a fixed τ = 0.05 corresponds to different dynamical scales at different b.

2.4 Diagnosis

m_eff(τ) is not an intrinsic parameter. It counts "how many steps of size τ fit inside a fixed feedback budget." In continuous-time oscillators, K should not be extracted via one-lag C(1).

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§3 Negative Result II: K_part Does Not Apply at Finite βE

3.1 Theoretical motivation

Thermo IV §3.5 proved a supplementary lemma: in the x → 0 second-order expansion,

$$q_{\text{eff}}^{(2)} = 1 + \sum_i \lambda_i^2$$

where λ_i are shielding-layer weights (Σλ_i = 1). This suggested an independent K-extraction scheme:

$$K_{\text{part}} = \frac{1}{\sum_i \lambda_i^2} \quad \text{(inverse participation ratio)}$$

3.2 Experiment: ACF mode decomposition

Fitting the Brusselator (b = 3.0) f-channel autocorrelation to a damped oscillation plus pure decay:

$C(\tau) = w_1 \cdot e\text{^}\{-g_1\tau\} \cdot \cos(\omega\tau) + w_2 \cdot e\text{^}\{-g_2\tau\}$

yields $w_1 = 0.477$, $w_2 = 0.523$ (normalized). $\Sigma w^2 = 0.477^2 + 0.523^2 = 0.501$.

K_part = 1/0.501 = 2.0. Predicted q = 1 + 0.501 = 1.501.

Measured q_fit = 1.091. **Deviation: 0.41.**

3.3 Why it fails

q_eff^{(2)} = 1 + $\Sigma\lambda^2$ is a second-order local expansion at x → 0. The Brusselator's effective βE ≈ 2.5–3.5 lies far from the small-x limit.

More fundamentally, this experiment is not a counterexample to Thermo IV §3.5, but a counterexample to the incorrect identification λ_i^{shield} = w_j^{ACF}. The two macroscopic ACF modes (oscillation + decay) are geometric rendering of the 4DD continuous flow; the λ_i in Thermo IV's theorem are microscopic weights of discrete shielding layers in the underlying feedback network. Eleven shielding layers are nested inside the internal structure of a single macroscopic oscillatory mode; spectral decomposition of the ACF cannot reach them.

3.4 Impact on Thermo IV

None. The supplementary lemma remains mathematically valid. The Brusselator experiment shows only that macroscopic ACF mode weights w_j cannot be identified with microscopic shielding-layer weights λ_i, especially outside the small-x regime. This is a boundary-condition test, not a counterexample.

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§4 Positive Result: The Channel-Normalized Winding Ratio

4.1 Extracting T and τ_dec from the ACF

The f-channel autocorrelation function is fitted to a damped oscillation plus pure decay:

$$C_f(\tau) = w_1 \cdot e^{-g_1\tau} \cos(\omega\tau) + w_2 \cdot e^{-g_2\tau}$$

Two macroscopic timescales are read directly from the fit parameters:

- **Oscillation period** T: from the first minimum of the ACF at t_min (giving T = 2 · t_min), or equivalently from the fitted angular frequency ω (giving T = 2π/ω). The data tables in this paper use the t_min method. At b = 3.0 the two methods differ by approximately 15% (t_min gives T = 5.76, ω gives T = 6.65), reflecting the non-sinusoidal character of the ACF oscillation under finite noise.
- **Radial decay time** τ_dec = $1/g_2$: the time constant of the pure-decay component extracted from the f-channel ACF. Note that τ_dec is not measured directly from phase-space radial regression, but is separated from the non-oscillatory term in the damped-oscillation fit.

Neither quantity depends on the sampling step τ — they are macroscopic dynamical parameters of the system.

4.2 Definition of K_dyn

Definition (channel-normalized winding ratio). In a state-coupled two-channel oscillator, the dynamical feedback order is defined as

$$K_{\text{dyn}} = \frac{T}{n_{\text{ch}} \cdot \tau_{\text{dec}}}$$

where n_ch = 2 is the number of forward/reset channels. The Tsallis q is predicted as

$$q_{\text{pred}} = 1 + \frac{1}{K_{\text{dyn}}} = 1 + \frac{n_{\text{ch}} \cdot \tau_{\text{dec}}}{T}$$

4.3 Physical interpretation

q − 1 = n_ch · τ_dec / T: the radial decay time as a fraction of the per-channel period budget.

The physical picture can be understood as time-division multiplexing of feedback bandwidth. The system possesses one macroscopic time resource — the oscillation period T — but n_ch channels must each process radial fluctuations. Because the dynamical operators associated with different channels do not commute (f and r share the state variable x), the underlying dynamics cannot execute causal settlement on multiple channels simultaneously. The system must therefore partition time: each channel is allocated an effective causal window of T/n_ch. Within this window, the rate at which the system erases a radial fluctuation is set by τ_dec. The effective shielding depth per channel per period is thus (T/n_ch)/τ_dec = K_dyn.

q − 1 measures the inverse of this per-channel depth — the fraction of each channel's time budget consumed by a single decay event.

4.4 Structural basis for n_ch = 2

The original hypothesis (proposed by Gemini) was $m_{eff} \approx T/\tau_{dec}$. Experiments showed that T/τ_{dec} yields an effective count approximately twice the required K. Applying Thermo IV's two-channel normalization $n_{ch} = 2$, the ratio $T/(n_{ch} \cdot \tau_{dec})$ produces substantially more robust predictions across the full b-scan (§5).

$n_{ch} = 2$ is not fitted from the present Brusselator data. It is a structural parameter of the Thermo IV framework, arising from the forward/reset channel decomposition. Thermo IV's channel-averaged shielding conjecture specifies that q measures per-channel shielding depth, so the total winding ratio T/τ_{dec} must be divided by the channel count.

4.5 Connection to Thermo IV

Thermo IV's exact structure $q = 1 + 1/K$ is fully preserved. This paper changes only how K is extracted:

	Thermo IV's K	This paper's K_dyn
Definition	Effective feedback order (conjecture)	$T/(n_{ch} \cdot \tau_{dec})$ (dynamically observable)
Extraction	Unspecified (listed as open problem)	From damped-oscillation fit of ACF
τ-dependence	—	None
Relation to m_eff	K "may equal" m_{eff} (conjecture)	$K_{dyn} \neq m_{eff}(\tau)$ in general

4.6 Status labels

Content	Level
$K_{dyn} = T/(n_{ch} \cdot \tau_{dec})$	**Empirical dynamical law / structurally motivated bridge**
$n_{ch} = 2$ from Thermo IV	Strong structural evidence, not proof
T and τ_{dec} from ACF	Operationally defined extraction protocol
$q = 1 + 1/K_{dyn}$	Exact if $K_{dyn} = K$

§5 Data: b-Scan

5.1 Experimental setup

Brusselator SDE [8]: $dx = (a + x^2y - (b+1)x)dt + \sigma dW\_x$, $dy = (bx - x^2y)dt + \sigma dW\_y$. Parameters: $a = 1$, $\sigma = 0.30$, $dt = 0.005$. Steady-state sampling: 1M steps. $r^2 = (x - \bar{x})^2 + (y - \bar{y})^2$ serves as the canonical energy-like observable. q_fit is extracted from the CCDF of r^2 via Tsallis q-exponential global fit.

Methodological note: The main results are reported using CCDF global fitting. As a robustification direction, raw-data MLE [9] and KS goodness-of-fit testing [10] should be implemented in follow-up work, with block-bootstrap 95% confidence intervals for pointwise uncertainty. CCDF fitting may be inferior to MLE for q-exponential parameter estimation [9], but is sufficient to support the MAE = 0.022 level of the present conclusions.

5.2 Results

b	T	τ_dec	K_dyn = T/(2τ_dec)	q_pred	q_fit(r^2)	Δq
2.2	5.58	0.288	9.70	1.103	1.110	+0.007
2.5	5.62	0.280	10.02	1.100	1.089	−0.011
2.8	5.68	0.278	10.22	1.098	1.085	−0.013
3.0	5.76	0.276	10.43	1.096	1.091	−0.005
3.5	6.07	0.263	11.54	1.087	1.103	+0.016
4.0	6.39	0.276	11.58	1.086	1.122	+0.036
5.0	6.26	0.349	8.97	1.112	1.179	+0.068

5.3 Error statistics

- $\text{MAE} = (1/7) \cdot \Sigma|\Delta q| = 0.022$
- $\text{RMSE} = \sqrt{(1/7) \cdot \Sigma(\Delta q)^2} = 0.031$
- Maximum $|\Delta q| = 0.068$ ($b = 5.0$)
- Core region ($b = 2.2$–3.5): maximum $|\Delta q| = 0.016$

5.4 Comparison with alternative methods

Method	Δq at b=3.0	Δq at b=5.0	τ-dependent?	Free parameters
Fixed τ=10 m_eff	0.001	0.215	Severely	1 (choice of τ)
T_fb method	0.048	—	Partially	0 but γ_f non-constant
K_part = $1/\Sigma w^2$	0.410	0.373	No	0 but x→0 fails
K_dyn = T/(n_ch·τ_dec)	**0.005**	**0.068**	**No**	**0**

K_dyn trades the 10^{-3} accidental precision at a single point (b = 3.0) for cross-parameter robustness. This is what good theory should do.

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§6 Methodology: Prior-Guided Posterior Verification

6.1 Prior basis for three key choices

The experimental design of this paper rests on prior theory published in Thermo III–IV. Three critical choices were determined by theory before the present experiments:

Observable (r^2): Thermo IV's §1.2 distinguishes kernel/occupation/flux layers, and §4 proves that the tail exponent is not a reparametrization invariant. r^2 is an energy-like variable corresponding to the canonical control variable βE. This is a theory-predicted canonical observable, not the result of trying multiple options.

Regime (moderate-noise σ = 0.10–0.30): Thermo III [3] independently established the Brusselator's moderate-noise absorptive regime — η ≈ 0.19–0.20 is most stable in this window. The coincidence of q's identification window with η's identification window is supporting evidence, not a consumed degree of freedom.

Estimator (global CCDF fit rather than tail-only): Thermo IV §4's reparametrization lemma proves that the tail exponent is not invariant under general reparametrizations. Choosing global fit over tail fit is a methodological consequence of Thermo IV, not a post-hoc selection.

6.2 Guarding against posterior colonization of priors

Prior-guided choices substantially reduce the researcher degrees of freedom in the present experiment, but do not entirely eliminate implementation-level freedom. The nature of a degree of freedom depends on its information source: choices derived from previously published theory are conceptual-level constraints (they do not consume the current experiment's degrees of freedom), but implementation choices such as CCDF vs MLE, fitting window, and bootstrap block size still exist.

	Unconstrained posterior fitting	**This paper**
Observable	Try several, pick the best	Theory-predicted canonical variable
Parameter window	Scan many, pick the prettiest	Prior paper established regime
Fitting method	Multiple methods, pick closest	Lemma excluded certain methods

The experimental design of this paper is a theory-constrained compatibility test with time-stamped priors, not unconstrained post-hoc selection. A strictly confirmatory test still requires pre-freezing the observable, regime, estimator, and error criteria on a second system.

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§7 Claim Boundary and Open Problems

7.1 Status map

Level	**Content**
Exact (from Thermo IV)	q = 1 + 1/K, exact interpolation family
Empirical dynamical law	K_dyn = T/(n_ch · τ_dec), channel-normalized winding ratio
Negative result	m_eff(τ) ∝ 1/τ is not intrinsic; K_part fails at finite βE
Structural evidence	n_ch = 2 consistent with Thermo IV's channel-averaged shielding
Open	Second-system verification; exclusionary test for n_ch; MLE + bootstrap

7.2 Main claim

In a state-coupled two-channel oscillator, the Tsallis excess q − 1 extracted on the canonical energy-like observable r^2 is predicted by the channel-normalized decay fraction n_ch · τ_dec / T, where τ_dec is extracted from the pure-decay component of the f-channel ACF:

$$K_{\text{dyn}} = \frac{T}{n_{\text{ch}} \cdot \tau_{\text{dec}}}, \quad q_{\text{pred}} = 1 + \frac{n_{\text{ch}} \cdot \tau_{\text{dec}}}{T}$$

This formula yields τ-free cross-parameter predictions (MAE = 0.022) across b = 2.2–5.0, substantially outperforming the fixed-τ proxy m_eff(τ). This paper does not claim K = m_eff(τ) as a scale-free identity; instead, it identifies K in continuous-time oscillators as a channel-normalized period–decay ratio.

7.3 Propositions beyond the scope of this paper

- Whether K_dyn applies to non-oscillatory systems — currently verified only on oscillators.
- Whether the factor 2 in n_ch = 2 truly arises from the f/r channel count — or from other geometric factors (half-period, r^2 frequency folding) — requires exclusionary tests on three-channel systems or single-channel models. The primary candidate for a three-channel test is the Lorenz attractor ($\dot{x} = \sigma(y-x)$, $\dot{y} = x(\rho-z)-y$, $\dot{z} = xy-\beta z$; three state variables → n_ch = 3), testing $q = 1 + 3\tau\_dec/T$. An alternative is the full three-variable Oregonator model.
- Whether $q\_pred = 1 + n\_ch \cdot \tau\_dec / T$ is exact — or a leading-order approximation requiring higher-precision data and/or additional systems.

7.4 Open problems

1. First-principles derivation of $K\_dyn = T/(n\_ch \cdot \tau\_dec)$ — from what physical principle does "K equals the channel-normalized winding ratio" follow?
2. K extraction in non-oscillatory systems (e.g., DP recursion, Schlögl model) — K_dyn's definition relies on an oscillation period T; what replaces it when T does not exist?
3. Exclusionary test for n_ch — verify $q = 1 + 3\tau\_dec/T$ on the Lorenz system (n_ch = 3), or verify that the factor 2 disappears in a single-channel model.
4. Source of high-b deviation — $|\Delta q| = 0.068$ at $b = 5.0$; does this arise from breakdown of the equal-sharing assumption under strong nonlinearity?
5. Reverse mapping from K_dyn to Ω_eff — from $q = \Omega\_eff/n\_ch$ and $q = 1 + n\_ch \cdot \tau\_dec/T$, one obtains $\Omega\_eff = n\_ch + n\_ch^2 \cdot \tau\_dec/T$. For the Brusselator (n_ch = 2, $T \approx 5.76$, $\tau\_dec \approx 0.276$), $\Omega\_eff \approx 2.19$ — close to the Boltzmann end. Does this kernel-level Ω_eff have an independent physical interpretation? What is its relation to the cycle-level winding budget $T/\tau\_dec \approx 20.9$?
6. Connection to the Lyapunov spectrum — τ_dec dynamically corresponds to the convergence rate in the transverse direction, i.e., $\tau\_dec \propto 1/|\lambda_-|$ where λ_- is the most negative Lyapunov exponent. Substitution yields $q = 1 + n\_ch/(T \cdot |\lambda_-|)$, directly linking the distribution shape parameter to the orbital convergence rate. Testing this relation on chaotic systems (e.g., Lorenz) is a longer-term direction.

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